QUESTION 2012

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following:

i) Integrating factor of the differential equation

$x(1-x^2)dy+(2x^2y-y-ax^3)dx=0$ is

a) $x/\sqrt{1-x^2}$ b) $x/\sqrt{x(x^2-1)}$ c) $x/\sqrt{x^2-1}$ d) $x^2/\sqrt{1-x^2}$

Answer: $\frac{1}{x\sqrt{1-x^2}}$

ii) The order and degree of the differential equation

$\sqrt{d^2y/dx^2}+dy/dx=y$ are

✓a) 2, 1 b) 1, 2 c) 2, 2 d) 2, 3

iii) $\frac{1}{(D-2)(D-3)}e^{2x}$ is

a) $-e^{2x}$ b) xe^{2x} ✓c) $-xe^{2x}$ d) $-xe^{3x}$

iv) If for a sequence (U_n), $\lim_{n\to\infty} U_n=0$, then

a) $\{U_n\}$ is convergent b) $\{U_n\}$ is divergent

✓c) $\{U_n\}$ is convergent to 0 d) none of these

v) The infinite series $\sum_{n=0}^{\infty}\frac{n}{n+1}$ is

a) divergent b) convergent c) oscillatory ✓d) none of these

vi) The value of *a* for which $\{(1,2,3),\ (0,-1,9),\ (4,0,a)\}$ is linearly dependent is

a) – 20 b) – 10 c) – 5 ✓d) None of these

vii) If the third order square matrix *A* is diagonalizable, then the number of independent eigenvectors of *A* is

a) two ✓b) three c) one d) none of these

viii) If S and T be two subspaces of a vector space V, then which of the following is also a subspace of V?

a) $S \cup T$ b) $S - T$ c) $T - S$ ✓d) $S \cap T$

ix) The dimension of the subspace $\{(x, 0, y, 0) \mid x, y \in R\}$ is

a) 1 ✓b) 2 c) 3 d) 4

x) Let V and W be two vector spaces over R and $T: V \to W$ is a linear mapping. Then $Im\ T$ is a sub-space of

a) V b) W c) $V \cup W$ ✓d) $V \cap W$

xi) The infinite series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if

a) $p \geq 1$ ✓b) $p > 1$ c) $p \leq 1$ d) none of these

xii) The lower bound of the sequence $\{(-1)^{n-1}/n!\}$ is

✓a) $-1/2$ b) $1/2$ c) 0 d) none of these

xiii) Eliminating A and B from $y = A\cos x + B\sin x$, the differential equation is

a) $\frac{d^2y}{dx^2} = 0$ b) $\frac{d^2y}{dx^2} - y = 0$ ✓c) $\frac{d^2y}{dx^2} + y = 0$ d) $\frac{d^2y}{dx^2} = 1$

xiv) The particular integral of $(D^2 + 1)y = \sin x$ is

a) $x\sin x$ b) $x\cos x$ c) $x\tan x$ ✓d) $-\frac{x}{2}\cos x$

GROUP – B
(Short Answer Type Questions)

2. Solve: $(px - y)(py + x) = a^2 p$ by using the substitution $x^2 = u$, $y^2 = v$ where $p = \frac{dy}{dx}$

See Topic: DIFFERENTIAL EQUATIONS, Long Answer Type Question No. 1(b).

3. Examine the convergence of the sequence $\left\{\left(1+\frac{2}{n}\right)^n\right\}$.

See Topic: SEQUENCE, Short Answer Type Question No. 6.

4. Examine the convergence of the series: $\frac{1}{2}+\frac{2}{3}x+\left(\frac{3}{4}\right)^2 x^2+\left(\frac{4}{5}\right)^3 x^3+\ldots\ldots\ldots\ldots$

See Topic: SERIES, Short Answer Type Question No. 11.

5. Show that $W=\{(x_1, x_2, x_3, x_4)\in R^4 \mid x_1-x_2+x_3=x_4\}$ is a subspace of R^4.

See Topic: LINEAR ALGEBRA, Short Answer Type Question No. 18.

6. Find the representative matrix of the linear transformation $T: R^3 \to R^3$ defined by

$$T(x, y, z)=(3x+z, -2x+y, -x+2y+4z)$$

See Topic: LINEAR ALGEBRA, Short Answer Type Question No. 19.

7. Find the basis of $S=\{(x, y, z)\in R^3 \mid x+2y+z=0,\ 2x+y+3z=0\}$

See Topic: LINEAR ALGEBRA, Short Answer Type Question No. 20.

GROUP – C
(Long Answer Type Questions)

8. a) Solve: $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}=12\frac{\log x}{x^2}$

b) Obtain the general solution and singular solution of the equation $y=px+\sqrt{a^2p^2+b^2}$

c) Solve: $3\frac{dy}{dx}+\frac{2y}{x+1}=\frac{x^3}{y^2}$

See Topic: DIFFERENTIAL EQUATIONS, Long Answer Type Question No. 10(a), 8(b) & 10(b).

9. a) State Leibnitz theorem for Alternating series and test the convergence of the series

$$1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots\ldots\ldots$$

b) Test the convergence of the following series

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{2^3}+\frac{1}{3^3}+\ldots\ldots\ldots\ldots$$

c) Show that the sequence $\left\{2+\frac{(-1)^n}{n}\right\}$ is convergent.

a) See Topic: SERIES, Long Answer Type Question No. 9(a).
b) See Topic: SERIES, Long Answer Type Question No. 9(b).
c) See Topic: SEQUENCE, Short Answer Type Question No. 4.

10. a) Solve $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+29y=0$ when $x=0,\ y=0,\ \frac{dy}{dx}=15$

b) Show that the sequence $\sqrt{2},\ \sqrt{2+\sqrt{2}},\ \sqrt{2+\sqrt{2+\sqrt{2}}},\ \ldots\ldots\ldots$ converges to 2.

c) Define basis and dimension of a vector space.

a) See Topic: DIFFERENTIAL EQUATIONS, Long Answer Type Question No. 11.
b) See Topic: SEQUENCE, Long Answer Type Question No. 5.
c) See Topic: LINEAR ALGEBRA, Short Answer Type Question No. 6.

11. a) Prove that the vectors (x_1, y_1) and (x_2, y_2) are linearly dependent, if and only if $x_1y_2-x_2y_1=0$

b) Show that the vectors $\alpha_1=(1,0,-1), \alpha_2=(1,2,1)$ and $\alpha_3=(0,-3,2)$ form a basis of R^3. Express $(1,0,0)$ as a linear combination of α_1, α_2 and α_3.

c) If $\alpha_1, \alpha_2, \alpha_3$ form a basis of a vector space V, then prove that $\alpha_1+\alpha_3, 2\alpha_1+3\alpha_2+4\alpha_3$ and $\alpha_1+2\alpha_2+3\alpha_3$ also form a basis of the vector space V.

a) See Topic: LINEAR ALGEBRA, Long Answer Type Question No. 5(c).
b) & c) See Topic: LINEAR ALGEBRA, Long Answer Type Question No. 14 (a) & (b).

12. a) Let T be defined by $T(x, y)=(x', y')$ where $x'=x\cos\theta-y\sin\theta,\ y'=x\sin\theta+y\cos\theta$. Prove that T is a linear transformation.

b) The linear transformation $T: R^3\to R^3$ transforms the basis vectors $(1,2,1),\ (2,1,0)$ & $(1,-1,-2)$ to the basis vectors $(1,0,0), (0,1,0)$ & $(0,0,1)$ respectively. Find T. Hence find $T(3,-3,3)$.

c) Find the Kernel, Image, Nullity and Rank of

$T: R^3\to R^2$ where

$T(1,0,0)=(2,1)$

$T(0,1,0)=(0,1)$

$$T(0,0,1)=(1,1)$$

a) & b) See Topic: LINEAR ALGEBRA, Long Answer Type Question No. 15(a) & (b).

c) See Topic: LINEAR ALGEBRA, Long Answer Type Question No. 5(a).

13. a) Prove that a subset S of a vector space V over R is a subspace if and only if $\alpha x+\beta y\in S$ for all $\alpha,\beta\in R$ and $x, y\in S$.

b) Show that the family M_2 of all real square matrices of order 2 forms a vector space over reals, and find a basis for M_2.

c) Let $S=\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a+b=0 \text{ and } a,b,c,d\in R\right\}$. Prove that S is a subspace of M_2.

See Topic: LINEAR ALGEBRA, Long Answer Type Question No. 16 (a), (b) & (c).